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The snow cannot have come from the pole, nor from any source beneath the cap. Snow descends from the atmosphere. It cannot have come from the cap itself; for that does not exist for about half the year. It must have had its origin from some body of water nearer the equator than itself. In any such case, the argument of the foregoing computation maintains its force; for the water of which it is composed must return to the place whence it came, else the supply would soon be exhausted.

So it would appear, at least to an amateur in the study of astronomy, such as the writer frankly confesses himself to be, that, when the material of the Martian snow-cap moves *from a position near the axis of rotation to one more remote from it, momentum must be lost.* As the counteracting influences all combined are insufficient to restore what has been taken away, axial velocity must be diminished, and the day on *Mars* must be slowly but surely growing longer.

PARK COLLEGE, PARKVILLE, MO., December, 1898.

THE CONSTANT OF ATTRACTION.

By FREDERICK H. SEARES.

In view of the confused designations of the constant of attraction as given by many writers upon theoretical astronomy, it is perhaps worth while for the benefit of students to call attention to its true significance.

NEWTON'S law of attraction states a proportionality between the force of attraction exercised by two bodies, on the one hand, and the product of their masses divided by the square of the distance between them, on the other. In practice it is convenient to express this law in mathematical symbols in the form of an equation, the proportionality being converted into an equality by the introduction of a constant factor whose numerical value depends upon the numerical values employed for the fundamental units. Assuming the law of gravitation to be accurately true for all parts of space, this factor, which may be called the constant of attraction, is a true constant of the universe. Its value changes only with a change in the fundamental units.

The law of attraction therefore takes the form

$$\text{Force} = A\ (\text{mass}) \times (\text{acceleration}) = \frac{C\ (\text{mass}) \times (\text{mass})}{(\text{distance})^2} \qquad (1)$$

where C is the constant under consideration, and A is a numerical quantity, usually unity, introduced to bring about an equality between the force and the product of the mass into the acceleration. Considered as an equation of dimensions, (1) is of the form

$$m\ l\ t^{-2} = C\ m\ m\ l^{-2}\ .\ .\ .\ .\ .\ .\ (2)$$

from which it appears that C is of the dimensions $m^{-1}\ l^3\ t^{-2}$, mass, length, and time being the fundamental units. It has upon occasion seemed desirable to express mass as a derived unit of the dimensions $l^3\ t^{-2}$, in which case C is an abstract number. With the introduction of this new conception of mass it is to be noted that either A must be thought of as a dimensional quantity or a new conception of force must be adopted. With the usual conception of mass, C is a dimensional quantity of the form $l^3\ t^{-2}$; with the new conception it is an abstract number; it is never a force or an acceleration, as is frequently stated, for it has the dimensions of neither of these quantities, although under certain circumstances it may become *numerically equal* to them. For example, if in (1) the masses and the distance are all unity, then C, the force, and the acceleration are all numerically equal. It is on account of this fact that confusion has arisen; and simple as the matter is, it is often something of a puzzle to the student who has arrived at this point in his advance.

The constant of attraction appears in two different classes of problems. First, in problems involving the relative motions of the members of the solar system. It is then called the Gaussian constant of planetary motion, and is denoted by the symbol k^2. Secondly, it appears in problems involving the forces, accelerations, etc., due to gravitation and weight, in small masses on or near the surface of the earth. In this connection, it is called the constant of gravitation, and is usually designated by the symbols k or G. The principle involved in the use of the constant is precisely the same in the two cases; the only difference is one of numerical value.

The theory of elliptical motion leads to the equation

$$k = \frac{2\ \pi\ a^{\frac{3}{2}}}{T\sqrt{M + m}}\ \ .\ .\ .\ .\ .\ .\ (3)$$

where M and m are the two attracting masses, a their mean distance, and T the period of revolution about their common center of gravity. This equation, combined with observations upon members of the solar system, gives the numerical value of the

Gaussian constant of planetary motion. As originally defined by GAUSS, its value depended upon the mass of the Sun as the unit of mass, the mean solar day as the unit of time, and the mean distance of the Earth from the Sun as the unit of distance. Since the original evaluation of the quantity by GAUSS, more accurate values have been obtained for both the mass of the Earth and its period of revolution about the Sun, and in the future doubtless still more accurate values will be found for these quantities. It is evident that if we are to adhere strictly to the best determinations of these quantities that changes must be made from time to time in k^2. But extensive tables have been computed with the original value of the constant, and any change in its value would lead to much inconvenience. It has seemed best, therefore, to retain unchanged the value of k^2 as given by GAUSS, and to vary the unit of distance as occasion demands, so that upon the introduction into (3) of improved values of m and T, the equation is still satisfied.

The value to be used for k is therefore

$$k = 0.017\ 202\ 098\ 95; \text{ or, } \log k = 8.235\ 581\ 441\ 4 - 10$$

and using

$$T = 365.256\ 357\ 4 \qquad m = 1:330\ 000$$

the resulting value of a is given by

$$\log a = 0.000\ 000\ 009\ 9$$

from which it follows that the corresponding unit of distance is 0.999 999 977 2 of the mean distance of the Earth from the Sun.

In this connection, see GAUSS'S *Theoria Motus*, First Book, First Section; also, V. OPPOLZER'S *Lehrbuch zur Bahnbestimmung der Kometen und Planeten*, Vol. I, p. 48.

The value of the constant as it appears in the second class of problems, depends ordinarily upon the centimeter-gram-second system of units. It is possible to determine this value directly from equation (3); it can also be obtained from the value of the GAUSSIAN constant by multiplying by the number which will make the transfer from the one system of units to the other; but a much more accurate method is afforded by equation (1) when applied to the results of experiments for the determination of the mass and mean density of the Earth. In this case we must measure the force expressed by $G\,\frac{m\,m_1}{d^2}$, the masses m and m_1 and the distance d being known. They may be the mountain and the plumb-bob, as in the experiment of MASKELYN, or the surface strata of the Earth and the plumb-bob, as in the experiment of AIRY, or two spheres of known mass, as in all the various forms of the CAVENDISH experiment. It may be remarked that the gravitation constant being known, the mean density of the Earth follows from the equation

$$G \frac{V \delta m_1}{R^2} = \text{weight of } m_1 = m_1 g$$

or,

$$\delta = \frac{g R^2}{G V}$$

where R and V are the radius and volume respectively of the Earth, and g the acceleration due to gravity. It is not necessary, however, that G be known in order to determine δ, for the experiment may be so varied as to eliminate G from the problem. The result of the most recent determination, that of Father BRAUN, is

$$G = 0.000\,000\,066\,552\,13$$

To recapitulate: The Gaussian constant, k^2, and the gravitation constant, k or G, are to be interpreted physically in exactly the same way; neither is ever force, mass, or acceleration; both are dimensional quantities of the form $m^{-1}\, l^3\, t^{-2}$, when, as is usual, mass, length, and time are the fundamental units. If mass be thought of as a derived unit of dimensions $l^3\, t^{-2}$, the constants are abstract numbers; but in any case their office is to convert into an equality the proportionality expressed by NEWTON'S law, and their only difference, which is a numerical one, arises from the difference in the system of units in the two classes of problems in which these constants occur.

The following are references to some of the more important papers treating of the determination of the mass and mean density of the Earth and the gravitation constant:—

V. JOLLY. *Abhandlungen d. 2. Kl. der Münch. Akad.* XIII. 1. Abth. S. 155–176. 1878: *Ann. d. Phys. u. Chem.* Neue Folge. XIV. S. 331–355. 1881: *Münch. Akad.* XIV. 2. Abth. S. 1–26. 1883.

Wiedemann's Ann. Bd. 5, S. 112: Bd. 14, S. 331.

POYNTING. *Proc. of the Royal Society of London*, XXVIII, p. 2, 1878.

Philosophical Transactions, vol. 182, p. 556.

Mean Density of the Earth; which is the Adams prize essay for 1893. GRIFFIN: London, 1894.

WILSING. *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, Nr. 22.

BOYS. *Philosophical Transactions*, vol. 186.

BRAUN. *Denkschriften der Kaiserlichen Akad. zu Wien*, Bd. 64.

Popular accounts and reviews of some of the above methods are to be found in

Nature, vol. 50, pp. 330, 366, 417; vol. 56, p. 127.

Himmel und Erde, Bd. X, p. 385.

Vierteljahrschrift der Astronomischen Gesellschaft, Bd. 24, p. 18.

POYNTING'S prize essay contains a valuable historical account of the subject.

ASTRONOMICAL OBSERVATIONS IN 1898.

MADE BY TORVALD KÖHL, AT ODDER, DENMARK.

VARIABLE STARS.

*Z Cygni.**

January	10:	Z = a.
	11:	id.
	22:	id.
	27:	< a. > b.
	29:	id.
February	19:	= c.
April	14:	invisible.
	26:	id.
May	7:	id.
August	14:	= a.
	18:	a little > a.
	21:	id.
	24:	> a.
September	5:	much > a.
September	10:	> a. < A.†
	22:	id.
October	3:	id.
	6:	almost = A.
	9:	< A.
	26:	= a.
November	1:	< a. > b.
	4:	= b.
	6:	id.
	22:	< d. > e.
December	3:	< e.
	13:	id.
	18:	id.

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* *Vide* the sketch in the *Publications* A. S. P., No. 48, page 69.

† The position of *A* (7. Magn.) referred to 26 *Cygni* is: $\Delta\alpha = +1^m\,52^s$

$\Delta\delta = +12'.1$